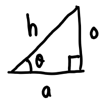
Lecture 2

Tuesday, September 6, 2016 8:52 AM

Trigonometric Identies

Appendix D : A28 - A30.

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$



- $\csc\theta = \dfrac{1}{\sin\theta}$
- $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$
- $\sec\theta = \dfrac{1}{\cos\theta}$

- $\sin^2\theta + \cos^2\theta = \left(\dfrac{o}{h}\right)^2 + \left(\dfrac{a}{h}\right)^2$

$$= \frac{o^2}{h^2} + \frac{a^2}{h^2} = \frac{o^2+a^2}{h^2} = \frac{h^2}{h^2} = 1$$

- $\boxed{\sin^2\theta + \cos^2\theta = 1}$ (*)

If you divide (*) by $\cos^2\theta$,

$\tan^2\theta + 1 = \sec^2\theta$

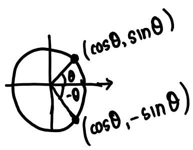
divide (*) by $\sin^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

- $\sin(-\theta) = -\sin\theta$ ← odd function

  $\cos(-\theta) = \cos\theta$ ← even function.

  } C.Y.



- $\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$.

  $\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$.

- $\sin(2x) = \sin(x+x) = \sin x \cdot \cos x + \cos x \cdot \sin x$

$$= 2\sin x \cdot \cos x$$

- $\cos(2x) = \cos(x+x)$

$$= \cos^2 x - \sin^2 x$$

- $\sin(x-y)$ *
- $\cos(x-y)$

  $\tan(x+y) = \dfrac{\sin(x+y)}{\cos(x+y)}$

  $\tan(x-y)$

  } DIY

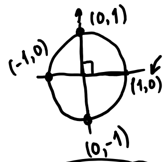
$\underline{\sin(x-y)} = \sin(x+(-y))$

$= \sin x \cdot \cos(-y) + \cos x \sin(-y)$

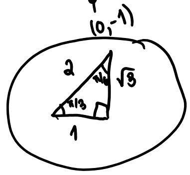
$= \sin x \cdot \cos y - \cos x \cdot \sin y$

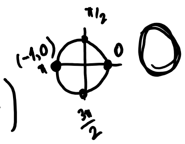
Graph of trigonometric functions:

$y = \sin x$



x	y
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1





$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)$

$= \cancel{\sin \pi}^{0} \cdot \cos\frac{\pi}{3} - \underset{-1}{\cos\pi} \cdot \sin\frac{\pi}{3}$

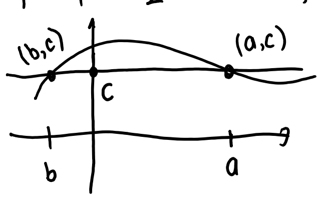
$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\sin x$ has a period of 2π

1.5 Inverse functions & Logarithms

Def A function f is called a 1-1 function if it never takes on the same value i.e.

if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$



$y = f(x)$

$f(a) = c$

$f(b) = c$

Not 1-1.

Horizontal Line Test

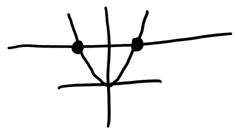
A function f is 1-1 if and only if no horizontal line intersects its graph at more than 1 point.

Ex $f(x) = x^2$ Not 1-1

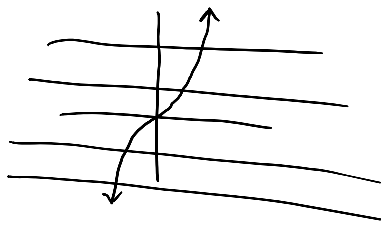
Ex $f(x) = x^2$ Not 1-1

$f(-1) = 1$

$f(1) = 1$



Ex $f(x) = x^3$



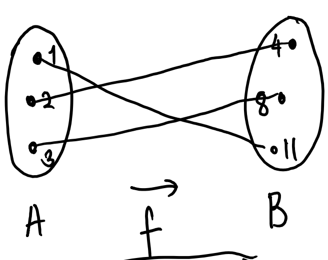
- If $x_1 \neq x_2$, then
  $x_1^3 \neq x_2^3$
  $f(x_1) \neq f(x_2)$.

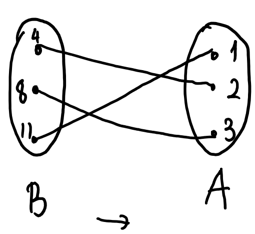
Why care about 1-1 function ?

DEF Let f be a 1-1 function w/ domain A and range B.
Then it's inverse function f^{-1} $\left(\neq \frac{1}{f}\right)$ has domain B and range A and is defined by :

$$f^{-1}(y) = x \text{ if and only if } f(x) = y.$$

for any y in B .





f^{-1}

$f(1) = 11$

$f^{-1}(11) = 1$

Again $f^{-1}(x) = y$

$\Updownarrow$

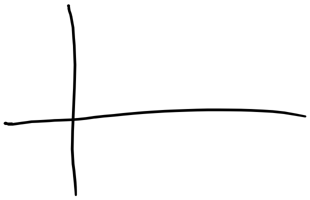
$f(y) = x$

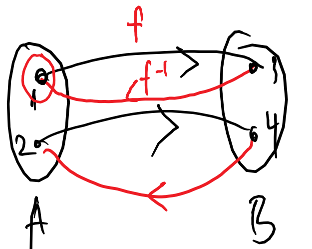
$$f^{-1}(f(1)) = f^{-1}(11) = 1$$

Cancellation Equations

$f^{-1}(f(x)) = x$ for every x in A

$f(f^{-1}(x)) = x$ for every x in B.





$f^{-1} \circ f(a) = a$